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Times of Execution of the CCNOT and CCCNOT Quantum Gates in a Quantum Computer based on a Quantum Dots Technology

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Abstract. It is considered a quantum computer consisting of a system of quantum dots (electrons) which are described by a Hamiltonian having rotation, inversion, and exchange symmetries. We consider exclusively the cases of both $n = 3$ and $n = 4$ quantum dots. In order to neglect their mutual Coulomb-like interactions, the electron dots are allocated very far way one of each other. In quantum dot technologies the figure of merit is the structure of the confinement potential. Thus, a realistic confinement potential of an electron dot which is composed of a very high rectangular potential well of width approximately equal to three orders of magnitude of the electron dot Compton wavelength (one nanometer) is proposed here. In order to verify the consistence of the present approach, the times of execution of the CCNOT and CCCNOT gates are calculated. It is found that in the case of a soft tunneling through the walls of the potential, the times of execution of such a gates are drastically small.

Keywords. Quantum dots, confinement potential, execution times, CCNOT gate, CCCNOT gate.

1 Introduction

Since the arising of the original proposal of quantum dots technology for quantum computation [1], a number of studies on the subject testing the consistence of the model have been made [2, 7]. It is worth emphasizing that a relevant technological application of quantum dots is the concerning to revolutionary TV displays [8-9]. In order to improve the technology it results important to study the gates which are a basic ingredients of this one.

So far the common quantum dots studies employed approaches have considered mainly systems of one and two electron dots. However, it is worth mentioning that in Refs. [5, 10] the study of systems composed by many-electron dots have also been included. In fact, in Ref. [5] it has been pointed out that many-body interaction arise when there are more than two electron spins in a computer. Through an approach based on an effective Hamiltonian, in Ref. [5] a system of three and four quantum dots were considered. From such an approach, important observations about contributions to decoherence from the many-body interactions have been done.

One of the main achievements of [5] is the calculation of the exchange interaction through the use of an harmonic oscillator confinement potential of the electron dots. However, such a confinement potential of an electron dot is not a realistic while in the original quantum dot model, the quantum dot confinement is almost complete and this includes scarce tunneling.

On the other hand, it is worth mentioning that the exchange interaction among three and four electron dots induces a CCNOT (Toffoli) [11] and CCCNOT gates respectively, however in Ref. [5] such an gates were not considered. In the present work we point out that the exchange interaction among three and four electron dots induces a CCNOT and CCCNOT gates which change one qubit if the each one of the rest of the control qubits have a value of one.

In order to be more realistic than Ref. [5], in the present approach it is employed a confinement potential of an electron dot which consist of a very

high potential well of approximated length equal to one nanometer[1].

With such a potential, we calculate the respective times of execution of CCNOT and CCCNOT gates as a function of the coupling constant between the spins. It is worth mentioning that the present approach is both a natural extension and refining of the work done in Ref. [5].

The Hamiltonian that we are employing is: $H = \sum_{i=1}^{n} \frac{\mathbf{p}_i^2}{2m} + V(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_n) + \sum_{i<j} \frac{e^2}{\kappa|\mathbf{r}_i - \mathbf{r}_j|}$, where $V(\mathbf{r})$ is the confining potential. The 2^n basis states are given by:

$$|\Psi(s_A, s_B, s_C, \cdots)\rangle = \sum_P \delta_P P[|AB...\rangle|s_A s_B ...\rangle],$$

where $|AB...\rangle$ refers to the orbital states of the electrons and $|s_A s_B ...\rangle$ are the spin of each electron. The sum includes all the $n!$ permutations being $\delta_P = 1$ (-1) if the permutation is even (odd) in such a way that the total eigenfunction is completely antisymmetric under exchange of an arbitrary pair of electron-dots.

If one assumes that the effective spin operator Hamiltonian (H_{spin}) has rotation, inversion, and exchange symmetry, considerable information is extracted from it. In particular, it can be concluded that this is a function of $\mathbf{S}_T^2 = (\mathbf{S}_A + \mathbf{S}_B + ...)^2$ where $\mathbf{S}_T$ is the total spin. It is worth mentioning that for a three quantum dots the following expansion of H_{spin} in powers of $\mathbf{S}_T^2$ has been essayed in [5]:

$$\begin{aligned} H_{spin} =& L_0 + L_1 \mathbf{S}_T^2 + L_2 (\mathbf{S}_T^2)^2 + ... \\ =& L_0 + \frac{9}{4} L_1 + 2L_1 \Big(\mathbf{S}_A \cdot \mathbf{S}_B + \\ & \mathbf{S}_A \cdot \mathbf{S}_C + \mathbf{S}_B \cdot \mathbf{S}_C \Big) \end{aligned} \tag{1}$$

where the expansion coefficients L_i $(i = 1, 2, ...)$ are real constants with dimensions of energy. The information on L_i can be extracted easily by employing the eigenstates of $\mathbf{S}_T^2$, that is:

$$H|\Psi_{S_T}^n\rangle = S_T(S_T + 1)|\Psi_{S_T}^n\rangle.$$

[1]The reason why the length of the confinement potential that we are employing here is approximately one nanometer is due that the scales of energy of the original quantum dot model are $\sim$ 10 MeV [1].

If we equate the two expressions $\langle\Psi_{S_T}^n|H_{spin}|\Psi_{S_T}^n\rangle = \langle\Psi_{S_T}^n|H|\Psi_{S_T}^n\rangle$ then it is possible to generate a $[\frac{n}{2}] + 1$ coupled linear equations for the coefficients L_i [5], where $[\frac{n}{2}]$ is the greatest integer less than $\frac{n}{2}$.

2 Proposal

2.1 Case $n = 3$ qubits

Through Eq. (1) and a simple manipulation of the results of Ref. [5] it is obtained that:

$$L_0 = \frac{5}{4} \cdot \frac{\epsilon_3 - \epsilon_0}{p_3 - p_0} - \frac{1}{4} \cdot \frac{\epsilon_3 + 2\epsilon_0 - 3\epsilon_1}{p_3 + 2p_0 - 3p_1}, \tag{2}$$

$$L_1 = \frac{1}{3} \cdot \frac{\epsilon_3 + 2\epsilon_0 - 3\epsilon_1}{p_3 + 2p_0 - 3p_1} - \frac{1}{3} \cdot \frac{\epsilon_3 - \epsilon_0}{p_3 - p_0}, \tag{3}$$

where the coefficients ϵ_i and p_i depend on the structure of the confinement potential.

In order to estimate the time of execution of the CCNOT (Toffoli) gate [2] it is necessary to calculate the transition frequencies between the states $|\uparrow\uparrow\uparrow\rangle$ and $|\uparrow\uparrow\downarrow\rangle$. For the above we need to know the following matrix elements of the Hamiltonian of Eq. (1):

$$\begin{aligned} \langle\uparrow\uparrow\uparrow|H_{spin}|\uparrow\uparrow\uparrow\rangle =& L_0 + \frac{9}{4} L_1 + 2L_1 \langle\uparrow\uparrow\uparrow| \\ & (\mathbf{S}_A \cdot \mathbf{S}_B + \mathbf{S}_A \cdot \mathbf{S}_C + \mathbf{S}_B \cdot \mathbf{S}_C)|\uparrow\uparrow\uparrow\rangle, \end{aligned} \tag{4}$$

$$\begin{aligned} \langle\uparrow\uparrow\downarrow|H_{spin}|\uparrow\uparrow\downarrow\rangle =& L_0 + \frac{1}{4} L_1 + 2L_1 \langle\uparrow\uparrow\downarrow| \\ & (\mathbf{S}_A \cdot \mathbf{S}_B + \mathbf{S}_A \cdot \mathbf{S}_C + \mathbf{S}_B \cdot \mathbf{S}_C)|\uparrow\uparrow\downarrow\rangle. \end{aligned} \tag{5}$$

In this way, the approximation for the execution time of the Toffoli gate is given by:

[2]The notation employed throughout the present work for the qubits is the following $|0\rangle \equiv |\downarrow\rangle$ and $|1\rangle \equiv |\uparrow\rangle$. Thus, the Hilbert space for the case of a 3-qubit state should be: $\{|\downarrow\downarrow\downarrow\rangle, |\downarrow\downarrow\uparrow\rangle, |\downarrow\uparrow\downarrow\rangle, |\uparrow\downarrow\downarrow\rangle, |\uparrow\uparrow\downarrow\rangle, |\uparrow\downarrow\uparrow\rangle, |\downarrow\uparrow\uparrow\rangle, |\uparrow\uparrow\uparrow\rangle\}$. The Toffoli gate is defined in such a way that this changes the 3-qubit state only when the two first (control) qubits are in the state $|\uparrow\rangle$. That is, CCNOT$|\uparrow\uparrow\downarrow\rangle \rightarrow |\uparrow\uparrow\uparrow\rangle$ and CCNOT$|\uparrow\uparrow\uparrow\rangle \rightarrow |\uparrow\uparrow\downarrow\rangle$, otherwise such a gate leaves unchanged the six remaining states of the Hilbert space.

$$\mathcal{T}_{CCNOT} \simeq \frac{2\pi\hbar}{|\langle\uparrow\uparrow\uparrow|H_{spin}|\uparrow\uparrow\uparrow\rangle - \langle\uparrow\uparrow\downarrow|H_{spin}|\uparrow\uparrow\downarrow\rangle|} = \frac{h}{|2L_1|}\Big(|2 + \langle\uparrow\uparrow\uparrow|(\mathbf{S}_A\cdot\mathbf{S}_B + \mathbf{S}_A\cdot\mathbf{S}_C + \mathbf{S}_B\cdot\mathbf{S}_C)| \uparrow\uparrow\uparrow\rangle - \langle\uparrow\uparrow\downarrow|(\mathbf{S}_A\cdot\mathbf{S}_B + \mathbf{S}_A\cdot\mathbf{S}_C + \mathbf{S}_B\cdot\mathbf{S}_C)|\uparrow\uparrow\downarrow\rangle|\Big)^{-1}, \quad (6)$$

where use has been made of Eqs. (4) and (5). In order to calculate the six matrix elements appearing in Eq. (6), we are using the well known result that the scalar product is invariant under rotations. The later means that $\mathbf{S}_i \cdot \mathbf{S}_j = S_i^z S_j^z$ which gives:

$$\begin{aligned}\mathcal{T}_{CCNOT} &\simeq \frac{h}{2L_1}\Big(|2 + \langle\uparrow\uparrow\uparrow|(S_A^z S_C^z + S_B^z S_C^z)|\uparrow\uparrow\uparrow\rangle - \\ &\quad \langle\uparrow\uparrow\downarrow|(S_A^z S_C^z + S_B^z S_C^z)|\uparrow\uparrow\downarrow\rangle|\Big)^{-1} \\ &= \frac{h}{6L_1}. \end{aligned} \quad (7)$$

In order to determine L_1, let us consider the following vertices of an equilateral tetrahedron contained in the XY plane $\mathbf{A} = (0,0,0)$, $\mathbf{B} = (\ell,0,0)$, $\mathbf{C} = (\ell,\ell,0)$, and $\mathbf{D} = (0,\ell,0)$ containing a quantum dot each of them. Furthermore, we are assuming that the present device is such that $\ell \gg 1$ in such a way that the mutual Coulomb interaction $\sum_{i<j}\frac{e^2}{\kappa|\mathbf{r}_i-\mathbf{r}_j|}$ between the electron-dots can be neglected.

From the above considerations, the Hamiltonian turns out to be: $H = \sum_{i=1}^n \frac{\mathbf{p}_i^2}{2m} + V(\mathbf{r}_i) + \sum_{i<j}\frac{e^2}{\kappa|\mathbf{r}_i-\mathbf{r}_j|} \simeq \sum_{i=1}^n \frac{\mathbf{p}_i^2}{2m} + V(\mathbf{r}_i)$.
In order to fix the structure of the confining potential $V(\mathbf{r})$ of the Hamiltonian H, we note that according with the quantum dots model [1], an electron-dot is almost completely confined to a circle of radius of the order of three orders of magnitude the electron's Compton wavelength $L \sim 10^{-9}\ m$ [12]. Let us observe that the potential employed in [5] does not account for a realistic strict confinement of the electron-dots.

By the above reason, in the present work we are employing a very high rectangular potential well of longitude $\sim L$ which has the form:

$$V(\mathbf{r}) = V_A(\mathbf{r})V_B(\mathbf{r})V_C(\mathbf{r})V_D(\mathbf{r}), \quad (8)$$

where

$$V_\xi(\mathbf{r}) = \begin{cases} 0 & \text{if } |\mathbf{r}-\xi| \le L \\ \frac{1}{2}m\omega_0^2 L^2 & \text{if } |\mathbf{r}-\xi| > L \end{cases}, \quad (9)$$

for $\xi = \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ and $\frac{1}{2}m\omega_0^2 L^2$ is the very large height of the potential. The respective energy levels of each dot are $E_{n_\xi} = \frac{h^2}{8mL^2}\cdot {n_\xi}^2$ ($n_\xi = 1,2,3,\cdots$) in such a way that the total energy of the three quantum dots is:

$$E^{(3)} = \sum_\xi E_{n_\xi} = \frac{h^2}{8mL^2}$$
$$(n_A^2 + n_B^2 + n_C^2) \ge 3\cdot\frac{h^2}{8mL^2}, \quad (10)$$

providing that the Coulomb interaction between the electron-dots is neglected. In the present work, we are considering only the case $E^{(3)} < \frac{1}{2}m\omega_0^2 L^2$ which accounts for the strict confinement of the electron-dots. On the other hand, the exchange interaction is proportional to the magnitude of the overlap between functions centered in different dots.

Consequently, with a confinement potential of a very high square well potential, the exchange interaction must be both small and proportional to the transmission coefficient through the walls. In other words, we are assuming that the exchange interaction $J = 2L_1$ is such that, it results simpler to calculate J in this way than with the use of Eq. (3):

$$J = (1-R)\,E^{(3)}, \quad (11)$$

where the reflection coefficient of the electron-dot against the wall is [13]:

$$R = 1 - T = \frac{\left(\kappa_1 - \kappa_2\right)^2}{\left(\kappa_1 + \kappa_2\right)^2}, \quad (12)$$

being
$\kappa_1 = \sqrt{2mE^{(3)}/\hbar^2}$,
$\kappa_2 = \sqrt{2m(m\omega_0^2 L^2/2 - E^{(3)})/\hbar^2}$.

and T is the transmission coefficient of the quantum dot electron through the walls of the potential. It results convenient to define the exchange energy J of Eq. (11) in units of the height of the potential well which is $\frac{1}{2}m\omega_0^2 L^2$. To assume that each electron-dot is in its ground state and by defining the dimensionless quantity $x_b \equiv E^{(3)}(n_A = 1, n_B = 1, n_C = 1)/(m\omega_0^2 L^2/2) = h^2/(4m^2\omega_0^2 L^4)$ in Eqs. (11) and (12) one obtains that:

$$j \equiv \frac{J}{m\omega_0^2 L^2/2} = 1 - 3x_b\left(\frac{\sqrt{3x_b}-\sqrt{3x_b-1}}{\sqrt{3x_b}+\sqrt{3x_b-1}}\right)^2, \qquad (13)$$

where $\frac{1}{3} \leq x_b$. In Figure 1 it is plotted the quantity j of Eq. (13) as a function of x_b in the ground state. From such a figure one can see that when the electron-dots become more excited (i.e. $x_b \to 1/3$), the exchange energy is close to unit but that if the electron-dots are approximately in their ground state then j vanishes. In the general case when the three electron dots are not in their ground states, from Eqs. (7, 10, 11) one can see that the respective values of the quantity j would correspond to the shaded region of Figure 1.

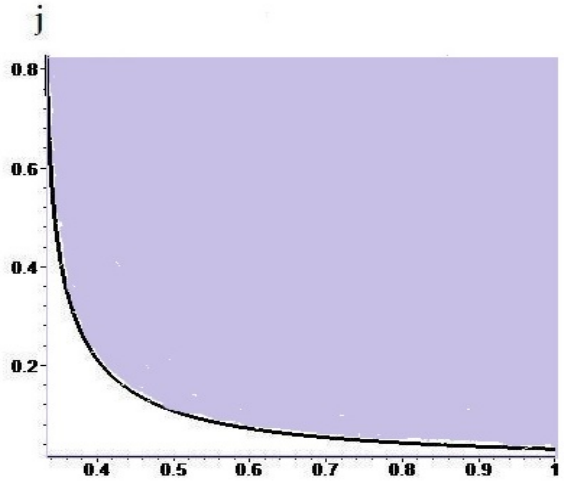

Fig. 1. The exchange energy of Eq. (13) as a function of x_b in the range $1/3 \leq x_b$. The curve corresponds to the ground state while the shaded region to the excited states of the three quantum dots

Substitution of (13) in (7) yields:

$$\mathcal{T}_{CCNOT} = \frac{1}{3}\cdot\left[1 - 3x_b\left(\frac{\sqrt{3x_b}+\sqrt{3x_b-1}}{\sqrt{3x_b}-\sqrt{3x_b-1}}\right)^2\right]^{-1} \cdot\left(\frac{h}{m\omega_0^2 L^2}\right). \qquad (14)$$

The above expression predicts that when the height of the box is very high i.e. $x_b \sim 1/3$ and $L \ll \ell$, the electron-dots are incommunicable and consequently the time of execution of the Toffoli gate is very large. On the contrary, if there is a soft tunneling through the box i.e. $x_b \sim 1$ the time of execution of the CCNOT gate is drastically small. This might indicate that the presence of an interaction between the three quantum dots is a necessary ingredient for the execution of the CCNOT gate.

On the other hand, in Ref. [14] it has been suggested that the decoherence times in quantum dots technologies are typically $\sim 10^{-7}\ s$. In Figure 1 is plotted $\mathcal{T}_{CCNOT}$ of Eq. (14) as a function of x_b and $T_0 \equiv \frac{h}{m\omega_0^2 L^2}$ in the range $1/3 \leq x_b$ and $0 \leq T_0 \leq 10^{-7}\ s$. From such a figure we can infer that for a non small values of x_b it is increased the decoherence of the quantum dots, producing with this a large values of the time of execution of the CCNOT gate.

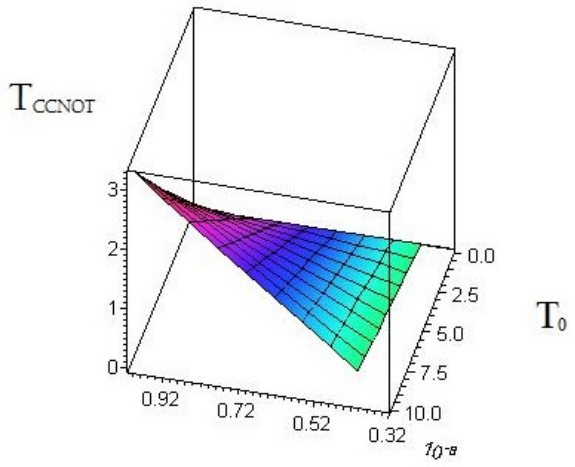

Fig. 2. Time of execution of the CCNOT gate as it is given by Eq. (14) as a function of x_b and T_0 in the range $1/3 \leq x_b$ and $0 \leq T_0 \leq 10^{-7}\ s$. The region below to the surface corresponds to the excited states of the system of three electron dots

On the other hand for small enough values of x_b and T_0, the above times of execution become small, making then the computer more efficient for processing the information. Ideally, one must have that $\mathcal{T}_{CCNOT} \ll 10^{-7}\ s$ which constrains the values of x_b and T_0 appearing in Eq. (14).

The scale of time $10^{-7}\ s$ is the estimated decoherence time of the electron-dots [14]. We note from Eqs. (7, 10, 11) that in the general case

when the three quantum dots are hyper excited i.e. $x_b \gg 1$, the values of $\mathcal{T}_{CCNOT}$ would correspond to the region below the surface of Figure 2.

The later can be explained by saying that when the quantum dots are excited then there will be tunneling effects allowing the necessary interaction between the three parties for executing efficiently the CCNOT gate diminishing with this its execution time.

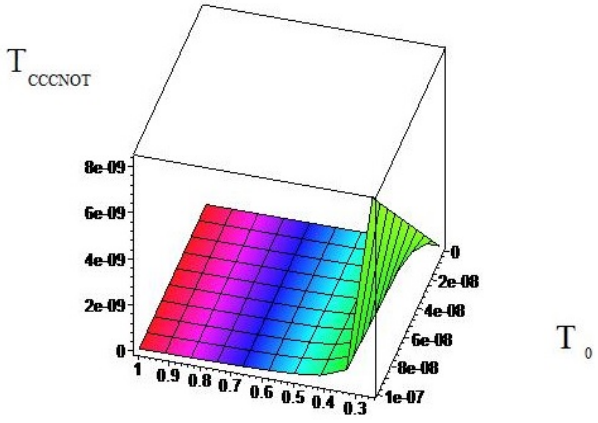

Fig. 3. Time of execution of the CCCNOT gate as it is given by Eq. (17) as a function of x_b and T_0 in the range $1/4 \leq x_b$ and $0 \leq T_0 \leq 10^{-7}\ s$. The region below to the surface corresponds to the excited states of the system of four electron dots

2.2 Case $n = 4$ qubits

The approximated expression for the time of execution of the CCCNOT gate [3] is related to the transition frequencies between the states $|\uparrow\uparrow\uparrow\uparrow\rangle$ and $|\uparrow\uparrow\uparrow\downarrow\rangle$ in the following way:

[3] In this case the respective Hilbert space is the following $\{|\downarrow,\downarrow,\downarrow,\downarrow\rangle, |\downarrow,\downarrow,\downarrow,\uparrow\rangle, |\downarrow,\downarrow,\uparrow,\downarrow\rangle, |\downarrow,\downarrow,\uparrow,\uparrow\rangle, |\downarrow,\uparrow,\downarrow,\downarrow\rangle, |\downarrow,\uparrow,\downarrow,\uparrow\rangle, |\downarrow,\uparrow,\uparrow,\downarrow\rangle, |\downarrow,\uparrow,\uparrow,\uparrow\rangle, |\uparrow,\downarrow,\downarrow,\downarrow\rangle, |\uparrow,\downarrow,\downarrow,\uparrow\rangle, |\uparrow,\downarrow,\uparrow,\downarrow\rangle, |\uparrow,\downarrow,\uparrow,\uparrow\rangle, |\uparrow,\uparrow,\downarrow,\downarrow\rangle, |\uparrow,\uparrow,\downarrow,\uparrow\rangle, |\uparrow,\uparrow,\uparrow,\downarrow\rangle, |\uparrow,\uparrow,\uparrow,\uparrow\rangle, \}$. The CCCNOT gate will change the states only if the three first (control) qubits are in the state $|\uparrow\rangle$ otherwise this leaves the remaining fourteen states unchanged. Thus, one has that CCCNOT$|\uparrow\uparrow\uparrow\downarrow\rangle \rightarrow |\uparrow\uparrow\uparrow\uparrow\rangle$ and CCCNOT$|\uparrow\uparrow\uparrow\uparrow\rangle \rightarrow |\uparrow\uparrow\uparrow\downarrow\rangle$.

$$\begin{aligned}\mathcal{T}_{CCCNOT} &= \frac{h}{|\langle\uparrow\uparrow\uparrow\uparrow|H_{spin}|\uparrow\uparrow\uparrow\uparrow\rangle - \langle\uparrow\uparrow\uparrow\downarrow|H_{spin}|\uparrow\uparrow\uparrow\downarrow\rangle|} \\ &= \frac{h}{24L_1} \\ &\equiv \frac{h}{12J'}. \end{aligned} \tag{15}$$

where H_{spin} is given by Eq. (1). Thus, if both the confinement potential of the four quantum dots is given by Eqs. (8) and (9) and we neglect the respective Coulomb interaction then the total energy of the four electron-dots system is given by:

$$\begin{aligned}E^{(4)} = \sum_{\xi} E_{n_\xi} = \frac{h^2}{8mL^2}(n_A^2 + n_B^2 + n_C^2 + n_D^2) \\ \geq 4 \cdot \frac{h^2}{8mL^2}. \end{aligned} \tag{16}$$

By assuming that $J' = (1-R)E^{(4)}$ where R and $E^{(4)}$ are given by Eqs. (12) and (16) respectively then Eq. (15) yields:

$$\begin{aligned}\mathcal{T}_{CCCNOT} &= \frac{h}{12(1-R)E^{(4)}} \\ &= \frac{1}{12} \cdot \left[1 - x_b\left(\frac{\sqrt{4x_b} + \sqrt{4x_b - 1}}{\sqrt{4x_b} - \sqrt{4x_b - 1}}\right)^2\right]^{-1} \cdot \\ &\quad \left(\frac{h}{m\omega_0^2 L^2}\right). \end{aligned} \tag{17}$$

In Figure 3 it is plotted $\mathcal{T}_{CCCNOT}$ as a function of x_b and T_0 in the range $\frac{1}{4} < x_b$ and $0 < T_0 < 10^{-7}\ s$. As it can be seen from such a figure, when the electron dots are excited there is tunneling which allows the presence of entanglement diminishing drastically the time of execution of the $\mathcal{T}_{CCCNOT}$ gate.

3 Conclusions

We have studied a systems composed of several quantum dots within an approximated architecture where the Coulomb-like interaction among them can be neglected. By the characteristics of the original electron dot model, the figure of merit is the nature of the confining potential. It is worth mentioning that in the original quantum dot model, the confinement is almost total with scarce tunneling towards outside the region of confinement.

For the above reason in the present work we employ a very high rectangular potential well. Due that the original quantum dot model of Loss and DiVincenzo works in the range of MeV, consequently we have assumed that the confinement potential has a length of the order of one nanometer. We remark that the exchange interaction between the quantum dots induces both CCNOT and CCCNOT gates.

Furthermore, in the present paper a soft tunneling is accounted to consider a not very excited states. Let us observe that a soft tunneling is necessary in order to execute a quantum logic gate. The above is due that with a soft tunneling of the electron dots, they can communicate each other through entanglement which is a necessary ingredient for an exponential speeding up of the processing of the information.

In such a situation the system of quantum dots can execute efficiently the respective quantum gates. The above argument is employed as a basis for the calculation of the times of execution of the CCNOT and CCCNOT gates. It is worth mentioning that the present estimation of such a times is not performed through the use of Eqs. (2) and (3) which involve a cumbersome calculations. Instead we employ both symmetry and probabilistic considerations given by Eq. (11). We have found that allowing a soft tunneling of the dot electrons through the walls of the potential the times of execution of both the CCNOT and CCCNOT gates are drastically small.

Acknowledgments

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